**Homework 7. Due on Wednesday, November 13**

1. Show that every polynomial of odd degree with real coeffients has at least one real root.

2. Let $f$ be continuous on the interval $[0,1]$ to $\mathbb{R}$ and such that $f(0) = f(1)$. Prove that there exists a point $c$ in $[0, \frac{1}{2}]$ such that $f(c) = f(c+\frac{1}{2})$. Conclude that there are, at any time, antipodal points on the earth's equator that have the same temperature.

3. Let $I := [a,b]$, and let $f : I \to \mathbb{R}$ be continuous on $I$ and assume that $f(a) < 0$, $f(b) > 0$. Let $S := \{x \in I : f(x) < 0\}$, and let $c := supS$. Prove that $f(c) = 0$.

4. Prove that $f(x) = \frac{1}{x^2}$ is uniformly continous on the set $A := [a, \infty)$ but not on the set $B := (0, \infty)$.

5. Problem 8 on page 99 of Rudin.

6. Problem 18 on page 100 of Rudin.

7. Problem 21 on page 101 of Rudin. (Assume the metric space $X$ is $\mathbb{R}$, and $K$ is a closed, bounded set in $\mathbb{R}$).

8. Problem 23 on page 101 of Rudin.