Homework 5. Due on Monday, November 4

1. Use the ϵ, δ definition to establish the following limits:
(a) $\lim_{x\to 2}(x^2+3x) = 10$.
(b) $\lim_{x\to c}\frac{1}{x} = \frac{1}{c}$. for any $c \neq 0$.
(c) $\lim_{x\to 1}\frac{x^2-x+1}{x+1} = \frac{1}{2}$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by setting $f(x) := x$ if x is rational, and $f(x) = 0$ if x is irrational.
(a) Show that f has a limit at $x = 0$.
(b) Show that if $c \neq 0$, then f does not have a limit at c.

3. Determine the following limit, justify your answer.
(a) $\lim_{x\to 0}\frac{(x+2)^2-4}{x}$
(b) $\lim_{x\to 0}\frac{\sqrt{(1+2x)}-\sqrt{(1+3x)}}{x+2x^2}$
(c) $\lim_{x\to 0} x cos(\frac{1}{x})$

4. Let f be continuous at c and $f(c) > 0$. Show that there exists a neighborhood of c such that f is positive on that neighborhood.

5. If $x \in \mathbb{R}$, define $[x]$ to be the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. The function $x| \to [x]$ is called the *greatest integer function.* Determine the points of continuity of the following functions:
(a) $f(x) := [x]$, (b) $g(x) := x - [x]$,
(c) $h(x) := x\,[x]$, (d) $k(x) := [\sin x]$.

6. Problem 1 on page 98 of Rudin.

7. Problem 2 on page 98 of Rudin. (You may take both the metric space X and Y to be the real line $\mathbb{R}$.)

8. Problem 16 on page 100 of Rudin.