Homework 3. Due on October 9

1. Find the interior points, the closure of the following sets; justify your answer:
(a) $E := \{\frac{1}{2^n} | n \in \mathbb{N}\}$
(b) $E := (\mathbb{Q} \cap (1,3)) \cup \{4\} \cup [5,6)$

2. Show that the intervals (a, ∞) and $(-\infty, a)$ are open sets, and that the intervals $[b, \infty)$ and $(-\infty, b]$ are closed sets.

3. Exhibit an open cover of $(1, 2]$ which does not have a finite subcover. Justify your example.

4. Let $E \not\equiv \emptyset$ be a closed and bounded set in $\mathbb{R}$ and let $c \in \mathbb{R}$. Prove that there exists a point $a \in E$ such that $|c - a| = inf\{|c - x| x \in E\}$.

5. Use the definition of the limit of a sequence to find the limit of the following:
(a) $lim\,(\,\frac{1}{\sqrt{n+3}}\,)$,
(b) $lim\,(\,\frac{n^2+5}{3n^2-2}\,)$
(c) $lim\,(\,\frac{1}{3^n}\,)$

6. Prove that $lim(x_n) = x$ then $lim(|x_n|) = |x|$, but the converse statement does not hold.

7. For x_n given by the following formulas, establish either the convergence or the divergence of the sequence $X = (x_n)$.
(a) $x_n := \frac{n}{n+2}$. (b) $x_n := \frac{(-1)^n n}{n+1}$. (c) $x_n := \frac{(-1)^n}{n+3}$.

8. Probelm 9 (a), (b) on page 43 of Rudin.

9. Problem 2, on page 78 of Rudin.

10. Problem 3, on page 78 of Rudin. What do you think is the limit of the sequence?