Homework 12. –Optional: due on or before January 10, 2003

1. Discuss the convergence and the uniform convergence of the series $\sum f_n$ where $f_n(x)$ is given by:
(a) $(x^2+n^2)^{-1}$, (b) $x^n(x^n+1)^{-1}$, $x \neq 0$.

2. Show directly that if $|x|<1$, the

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

3. Show by integrating the series for $\frac{1}{1+x}$ that if $|x|<1$, then

$$log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n.$$

4. Prove by introduction that the function given by $f(x) = e^{-\frac{1}{x^2}}$ for $x \neq 0$, $f(0)=0$, has derivatives of all orders at every point and that all of these derivatives vanish at $x=0$. Hence this function is not given by its Taylor series expansion about $x=0$.

5. Show that if $x>0$ and if $n>2x$, then

$$|e^x - (1+\frac{x}{1!}+\frac{x^2}{2!}+.....+\frac{x^n}{n!})| < \frac{2x^{n+1}}{(n+1)!}$$

Use this formula to show that $2\frac{2}{3} < e < 2\frac{3}{4}$, hence e is not an integer.

6. Problem 5 on page 166 of Rudin.

7. Problem 6 on page 166 of Rudin.

8. Problem 9 on page 166 of Rudin.

9. Problem 11 on page 167 of Rudin.