THE OPTIMAL FLUX FOR THE 'QUARTER-FILLED BAND'. Consider a large square lattice Λ with $|\Lambda|$ sites and with periodic boundary conditions. We assume that $|\Lambda|$ is divisible by 4. Now consider the hopping matrix T on Λ, but with a magnetic field, i.e., $T_{x,y} = \exp[i\theta(x,y)]$ if x, y are neighboring sites of Λ and $T_{x,y} = 0$ otherwise. The real numbers $\theta(x,y)$ are arbitrary, except for the hermiticity condition that $\theta(x,y) = -\theta(y,x)$. Let $\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq ... \leq \lambda_{|\Lambda|}$ denote the eigenvalues of T. It is easy to prove that these eigenvalues depend only on the fluxes, namely the sum of the $\theta(x,y)$ around the elementary small squares (or plaquettes) of Λ. With $E = \sum_{j=1}^{|\Lambda|/4} \lambda_j$ denoting the sum of the lowest one quarter of the eigenvalues, our goal is to find the choice of fluxes that minimize E. It has been conjectured that the optimum choice is when the flux through each elementary square is $\pi/2$. (Note: The corresponding problem for the half-filled band is known to have the optimum flux $= \pi$, even when a Hubbard type on-site interaction is included; E. H. Lieb, Phys. Rev. Lett. *73*, 2158-2161 (1994).)