BOSE-EINSTEIN CONDENSATION

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Recent experiments on dilute Bose gases of atoms seem to corroborate a long held belief that Bose-Einstein condensation occurs in weakly interacting systems. The Hamiltonian for such a gas of N particles can often be modeled by

$$H_N = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\Delta_j + \sum_{1\leq ij\leq N} W(x_i - x_j)$$

with a repulsive pair potential $W \geq 0$. If $W \equiv 0$ then the normalized symmetric ground state (with periodic boundary conditions in a large box of volume V) is $\Psi(x_1, ..., x_N) = V^{-N/2}$ and hence satisfies

$$C_\Psi := \int \Psi(x_1, x_2, ..., x_N)\Psi(y_1, x_2, ..., x_N)dx_1 dy_1 dx_2 dx_3 \cdots dx_N = V.$$

Bose-Einstein condensation in the ground state, with interaction W, is the statement that C_Ψ, defined as above, satisfies $C_\Psi \geq aV$ with $a > 0$, independent of V.

Can this be proved? It can be proved to be true for a gas of point hard core bosons on a hypercubic lattice in dimension 2 or more, but only for density 1/2. Even the extension of that result to other densities seems to be a hard problem. See E. H. Lieb, T. Kennedy and S. Shastry, Phys. Rev. Lett. **61**, 2582 (1988).

Some background can be found in E. Lieb, The Bose Fluid, *Lectures in Theoretical Physics, Vol. VIIC,* (Boulder summer school), University of Colorado Press, 175-224 (1965).

See also `http://amo.phy.gasou.edu/bec.html/bibliography.html#extheory`