Please write out the honor pledge and sign it:

NAME (print): __________________________

MAT 203 – Quiz 4
Due April 13, 2011

Information
Please read and sign the exam conditions first before turning the page:

• No books / notes / calculators / collaborations are allowed.
• The quiz has to be completed in a single time stretch of 90 min. No interruptions!
• Hand-in is on Wednesday, Apr. 13 in class. Please STAPLE your answer sheets, with this problem sheet as the front page. Write your full name legibly on every sheet.

I have read these conditions and will follow them (initials): ____________

Score:
Problem 1: ____________ points
Problem 2: ____________ points
Problem 3: ____________ points
Problem 4: ____________ points
Problem 5: ____________ points
1. Problem (12 points)

For the body

\[ B = \{(x, y, z) : x^2 + y^2 + z^4 \leq 1, z \geq 0\} \]

with uniform density one, calculate

(i) the total mass,

(ii) the center of mass,

(iii) the moment of inertia about the z-axis, and

(iv) the moment of inertia about the vertical line defined by \( x = 1 \) and \( y = 0 \).

2. Problem (6 points)

Consider the parametric curve

\[ c : [1, 2] \to \mathbb{R}^2 : t \mapsto (t, t^{-1}). \]

(i) Evaluate the path integral \( \int_c \sqrt{1 + y^4} \, ds \).

(ii) Evaluate the line integral \( \int_c (x^{-1}, y^{-1}) \cdot ds \).

3. Problem (8 points)

Consider the transformation

\[ T : \{(u, v) \in \mathbb{R}^2 : u > 0, v > 0\} \to \{(x, y) \in \mathbb{R}^2 : x \geq 2y > 0\} : \]

\[ (u, v) \mapsto (x, y) = (u^2 + v^2, uv). \]

(i) Is \( T \) one-one? Is \( T \) onto?

(ii) If we replace the domain of \( T \) with

\[ \{(u, v) \in \mathbb{R}^2 : u > v > 0\}, \]

is \( T \) then one-one? Is it then onto?

4. Problem (12 points)

(i) Determine if the improper double integral

\[ \iint_{x>y>0} e^{-x} \, dA \]

exists. Evaluate it if it does exist.
(ii) Let $\alpha > 0$ be any constant. Determine if the iterated improper integral
\[
\int_0^\infty \int_y^\infty e^{-x} \cos\left(\frac{\alpha y}{x}\right) \, dx \, dy
\]
exists. Evaluate it (in terms of $\alpha$) if it does exist.

(If your evaluations involve any abstract theorems, remember to verify the conditions needed.)

5. Problem (4 points)

Find the volume of
\[
E = \{(x, y, z) : x^2 + xy + y^2 + yz + z^2 \leq 1\}.
\]