Please write out the honor pledge and sign it:

NAME (print): ____________________________

MAT 203 – Quiz 2
Due February 28, 2011

Information
Please read and sign the exam conditions first before turning the page:

• No books / notes / calculators / collaborations are allowed.

• The quiz has to be completed in a single time stretch of 60 min. No interruptions!

• Hand-in is on Monday, Feb. 28 in class. Please **STAPLE** your answer sheets, with this problem sheet as the front page. Write your full name legibly on every sheet.

I have read these conditions and will follow them (initials): ______________

Score:

Problem 1: ____________ points
Problem 2: ____________ points
Problem 3: ____________ points
Problem 4: ____________ points
1. Problem (12 points)
Mark T (True) or F (False) in each of the statements. No justifications are required.

i) T F Every critical point of \( f(x, y) = x^2 - 4xy + x^4e^x \) is of saddle type.

ii) T F No critical point of \( f(x, y) = -3x + x^3 + y^2 \) is a local minimum.

iii) T F The function \( f(x, y, z) = xyz \) attains its maximal value over the set \( D = \{(x, y, z) : x^2 + y^2 + 2z^2 \leq 3\} \) at some boundary point(s) of \( D \).

iv) T F The function \( f(x_1, x_2, x_3, x_4) = x_1x_3 - x_2^2x_4 \) increases in the direction \((0, 1, 2, 3)\) at the point \((1, 1, 1, 1)\).

2. Problem (6 points)
Suppose \( f(x, y) \) is a \( C^2 \) function satisfying the partial differential equation \( f_{xx} + f_{yy} = 0 \). Verify that the function \( g(u, v) = f(u^2 - v^2, 2uv) \) satisfies the similar equation \( g_{uu} + g_{vv} = 0 \).

3. Problem (12 points)
Consider the curve parameterized by \( c(t) = (\cos t, \sin t, t^3) \) and the surface described by \( z^3 = 2 - x^2 - y^2 \) in \( \mathbb{R}^3 \).

i) Find the point \( P \) where the curve and the surface intersect.

ii) Find an equation for the tangent line \( L \) to the curve at \( P \).

iii) Find an equation of the form \( ax + by + cz = d \) (where \( a, b, c, d \) are constants) for the tangent plane \( E \) to the surface at \( P \).

iv) Let \( \theta \) be the intersection angle between \( L \) and \( E \). Calculate \( \sin \theta \).

4. Problem (6 points)
Let
\[
f(x, y) = \begin{cases} 
  \frac{x^2y^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\
  0, & \text{if } (x, y) = (0, 0)
\end{cases}
\]

i) Write down the linear approximation of \( f(x, y) \) at \((0, 0)\).

ii) Determine if \( f(x, y) \) is differentiable at \((0, 0)\).