

Name _____ Class Time _____

MATH 202 - QUIZ # 2
Wednesday, December 1, 2010
Covers Sections 5.3-5.4, 6.1-6.3 and 7.1-7.2 of the text
Time: 45 minutes

Please show all work. Unsupported answers will receive no credit. Books, notes, calculators, are not permitted on this quiz. As part of your obligations under the Honor Code, do not discuss this quiz with anyone until Friday's class.

WRITE OUT AND SIGN THE PLEDGE:

I pledge my honor that I have not violated the Honor Code during this examination.

1. (10 points) Consider $A = \begin{bmatrix} 8 & -15 \\ 2 & -3 \end{bmatrix}$.

(a) Find the eigenvalues of A .

(b) For each eigenvalue of A find a corresponding eigenvector \vec{v} .

(c) If $\vec{x}(t+1) = A\vec{x}(t)$, $x_1(0) = 1$ and $x_2(0) = 0$ then find $x_2(10)$.

2. (10 points) Let W denote the subspace of \mathbf{R}^4 defined by the equations

$$x_1 - x_2 + x_3 = 0 \quad \text{and} \quad x_1 - x_4 = 0.$$

Let V denote the orthogonal complement of W .

(a) Find a matrix A so that V is the image of A .

(b) Find the orthogonal projection of $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ onto V .

(c) Is the point $(1, 1, 1, 1)$ in \mathbf{R}^4 closer to V or to W ? (Justify your answer of course.)

3. (10 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 2 & 4 & 2 & 2 \\ 3 & 6 & 1 & -1 \\ -1 & 1 & -3 & 0 \end{bmatrix}$$

- (a) Compute the determinant of A .
- (b) Let \vec{v}_1 and \vec{v}_2 denote the first two columns of the matrix A . What is the area of the parallelogram in \mathbf{R}^4 spanned by \vec{v}_1 and \vec{v}_2 ?
- (c) What is the area of the parallelogram spanned by $2\vec{v}_1 - \vec{v}_2$ and $\vec{v}_1 + 3\vec{v}_2$?

4. (10 points) As usual, justify your answers.

(a) **T or F?** If A is a 4×4 matrix whose determinant is -5 then the matrix Q of the QR -factorization of A must have determinant -1 .

(b) **T or F?** If A is an $n \times n$ orthogonal matrix then all its n eigenvalues must be either 1 or -1 .

(c) **T or F?** If A is a singular (i.e. not invertible) matrix of the form $\begin{bmatrix} 1 - a & b & 1 \\ c & a & 2 \\ 3 & b & 1 \end{bmatrix}$, then either $a = 2b$ or $a = -2$.