Name ______________________________ **Class Time** ______________

MATH 202 - QUIZ # 1
Wednesday, October 6, 2010
Covers Chapters 1 and 2 of the text
Time: 45 minutes

Please show all work. Unsupported answers will receive no credit. Books, notes, calculators, are not permitted on this quiz. As part of your obligations under the Honor Code, do not discuss this quiz with anyone until after 1:30PM.

WRITE OUT AND SIGN THE PLEDGE:
I pledge my honor that I have not violated the Honor Code during this examination.

1. *(10 points)* Consider A, the matrix of orthogonal projection onto the line L in $\mathbf{R}^2$ defined by the equation $5x_1 = 12x_2$.

   (a) Find A.

   (b) If $B = A^2 + A - I_2$, then interpret B geometrically.

2. *(10 points)* Consider a 2×2 matrix A where

$$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \text{ and } A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

(a) Find A.

(b) Find A^{10}. (Hint: Think geometrically.)

3. *(8 points)* Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 \\ -1 & 2 & k^2 - 12 & k \end{bmatrix}$$

(a) What is the smallest rank that A can have? Which values of k give that rank?

(b) For what values of k, if any, will the matrix A be invertible?

4. *(12 points)* Determine whether the given statement is true or false. If the statement is always true, then explain why. If the statement is (at least sometimes) false, then give an example to show this. You will be graded on the clarity and completeness of your answers.

(a) If A is a rotation matrix that commutes with the matrix of reflection across the line $x_1 = x_2$ in $\mathbf{R}^2$ then $A = \pm I_2$.

(b) If A is a 6×3 matrix of rank 3 and if C is any matrix satisfying $AC = 0$, then all the entries of C are zero.

(c) There exists an invertible $n \times n$ matrix A where the sum of all the columns of A is the zero vector.

(d) If $T(\vec{x}) = A\vec{x}$ is a transformation from $\mathbf{R}^3$ to $\mathbf{R}^3$ where

$$T(\vec{e}_1) = \vec{e}_2, \quad T(\vec{e}_2) = \vec{e}_3 \quad \text{and} \quad T(\vec{e}_3) = \vec{e}_1$$

then $T^2 = T^{-1}$.