1. Find the points on the surface $2z^2 = 3xy + 8$ that are closest to/furthest from the origin.

2. The surface $S$ is the band on the sphere $x^2 + y^2 + z^2 = 1$ that satisfies $0 \leq z \leq \frac{1}{2}$. It is made of a material with uniform density. Find the center of gravity of $S$.

3. Consider the function $f(x, y, z) = x + y + z$.
   a) What is the directional derivative of $f$ at a point $(x_0, y_0, z_0)$ on the unit sphere $x^2 + y^2 + z^2 = 1$ along the unit outer normal vector of the sphere?
   b) Describe in words and by equation the set of points where this directional derivative zero.
   c) At which points of the unit sphere does such a directional derivative attain its maximum/minimum?

4. An airplane is flying along the path $(\cos t, \sin t, 2 - \cos t)$ (where $0 \leq t \leq 2\pi$) and carrying a laser pointing straight ahead.
   a) Sketch the path of the airplane.
   b) At what times does the laser beam shine on a point on the $(x, y)$-plane.
   c) The laser bean lights up a curve on the $(x, y)$-plane. Describe this curve in words and by an equation.

5. a) Graph and describe the surface $S_1$ given by $x^2 + y^2 = x$ and the surface $S_2$ given by $x^2 + y^2 + z^2 = 1$.
   b) Compute the volume of the region that lies inside both of these surfaces.

6. Consider the three–dimensional region $D$ given by $x^2 + y^2 \leq z^2 + 1$ and $x^2 + y^2 + 3z^2 \leq 5$.
   a) Describe in words and sketch $D$.
   b) Set up integrals (including appropriate limits) to integrate a function $f(x, y, z)$ on $D$ in cylindrical coordinates.

7. Homer Simpson’s favorite food is given in spherical coordinates by the equation $\rho \leq \sin \phi$. (Well, this doughnut is a little misshapen.) After he takes several bites, only the part of the doughnut in the first octant remains. Let $S$ be the surface which is the frosted (or exterior) part of this partial doughnut.
   a) Sketch the surface $S$. 
b) Write down a parametrization of $S$ using
   i) $\phi, \theta$ of the spherical coordinate system and
   ii) $r, \theta$ of the cylindrical coordinate system.

c) Describe the boundary curve of the surface $S$ and write down a parametrization of it.

8. Consider the surface $S$ cut out from the hemisphere $H : x^2 + y^2 + z^2 = 4, z \geq 0$, by the cylinder $Q : x^2 + y^2 = 2x$. Let $C$ be the boundary of $S$, oriented counterclockwise when viewed from the top.

   Compute the circulation of $F = -yi + xj + k$ along $C$.

9. Let $F$ be the vector field $(x - x^3)i + (y - y^3)j$.
   a) Find the closed curve $C$ in the plane such that the outward flux
      $\int_C F \cdot n\, ds$ is the largest possible. (Hint: Think! Very little computation is needed.)
   b) Compute this largest value.