On some problems in pointwise ergodic theory
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Abstract

Rosenblatt–Wierdl’s conjecture asserts that there is no sequence 
$(a_n)_{n \in \mathbb{N}} \subseteq \mathbb{N}$ with gaps tending to infinity, i.e. $a_{n+1} - a_n \to \infty$ for which

$$Mf(x) = \sup_{N \in \mathbb{N}} \frac{1}{N} \left| \sum_{n=1}^{N} f(x - a_n) \right|, \quad \text{for } x \in \mathbb{Z},$$

is of weak type $(1,1)$. The aim of the talk will be to show that there is a wide class of sequences of the form $a_n = \lfloor h(n) \rfloor$ for appropriate functions $h$, which give a negative answer for Rosenblatt–Wierdl’s conjecture. As a consequence we also obtain pointwise ergodic theorems along these sequences.

For instance, one can think that the function $h$ has the following form

$$h_1(x) = x^c \log^5 x, \quad h_2(x) = \frac{x^c}{e^{\log^{1/3} x}}, \quad h_3(x) = x^c \log \log x,$$

where $c \in (1, 30/29)$.

If the time allows we also define a counterpart of $Mf(x)$ along the set of prime numbers $P$. Namely

$$M_{P_h} f(x) = \sup_{N \in \mathbb{N}} \frac{1}{|P_h \cap [1,N]|} \left| \sum_{p \in P_h \cap [1,N]} f(x - p) \right|, \quad \text{for } x \in \mathbb{Z},$$

where

$$P_h = P \cap \{ \lfloor h(n) \rfloor : n \in \mathbb{N} \},$$

and we prove that $M_{P_h}$ is bounded on $\ell^r(\mathbb{Z})$ for all $r > 1$. 