Independent work by undergraduates in mathematics comes in many varieties. It may consist of an exposition of a novel approach to, or perspective on, a particular area or problem in pure or applied mathematics. In rarer cases, it may consist of new theorems, conjectures, computations, or applications that were previously unknown. In all cases, the emphasis is placed on the ability to delve deeply into a mathematical area, and to communicate mathematical ideas in this area with rigor, motivation, clarity, and logical reasoning.

The formal independent work requirements in the mathematics department consist of a junior seminar and/or junior paper during each term of the junior year, and a senior thesis in the final year.

Table of Contents

- Junior Independent Work ........................................................................................................... 2
  - Junior Seminars .......................................................................................................................... 2
  - Junior Papers ............................................................................................................................. 3
- Senior Theses ................................................................................................................................ 4
- Finding an adviser .......................................................................................................................... 5
- Timeline and Important Deadlines .............................................................................................. 6
  - Junior Seminars and JP's .............................................................................................................. 6
  - Senior Theses ............................................................................................................................. 6
- Evaluation of Independent Work .................................................................................................. 7
- Some suggestions for writing a good mathematics paper ............................................................. 9
- Some suggestions for giving a good mathematics lecture ............................................................. 9
- Resources for independent work .................................................................................................. 11
Junior Independent Work

All mathematics majors are expected to take a junior seminar during at least one semester of their junior year.

Junior Seminars

The purpose of the junior seminar (besides learning an interesting bit of mathematics not usually taught in courses) is to learn and experience the art of mathematical exposition, i.e. how to speak and write mathematics in a way that is accessible, clear, and interesting to others. Communicating mathematics can be just as important as doing mathematics! The junior year is an ideal time to learn and practice these skills.

Finding and using mathematical references to compile relevant material for a talk or paper is also a skill that students will get experience with while participating in a junior seminar.

Junior seminar topics for the forthcoming year will be announced at the end of the spring semester, and an organizational meeting will be held at the start of each semester, at which time students formally choose a seminar in which to participate.

A junior seminar typically meets about once a week for about 2 hours. After introductory lectures by the seminar leader in the initial couple of meetings, the lectures are then turned over to the student participants.

Presentations

Each seminar generally consists of two 45-minute lectures by a pair of students. It is recommended that the two students plan their lectures together and give practice talks to each other to ensure that the lectures are clear and accessible to the rest of the class. A day or two before the presentation the pair meets also with the seminar leader to share notes and do (at least a partial) practice run-through on the blackboard, to get additional feedback and suggestions for fine-tuning.

A student would generally participate in two such presentations during the term, with different partners.

Writing

About a month before the university deadline for submission of junior independent
work (usually the day after the start of reading period), the seminar leader will suggest a list of possible topics for a final short paper of 5-8 pages. Each participant will work independently on this final paper, on a topic of the participant’s choosing. Papers should be of interest and accessible to the participants of the seminar, requiring no further background beyond the seminar material.

The process of editing the paper is similar to that of preparing the talks. Each participant is placed in a pair of participants, who then read and criticize each other’s papers until they are clear, comprehensible, and interesting.

Finally, about a week before the due date, the participants give a mostly-polished draft to the seminar leader for any further comments. The final product is due by the university deadline for junior independent work.

**Junior Papers**

During one term of their junior year, students may choose to write a junior paper in lieu of a junior seminar. If deciding to do a junior paper, students should make an arrangement with an adviser about a topic and plan for their project and notify the undergraduate office by the end of the second week of the term in which they are writing the junior paper. Junior papers are due by the university deadline for junior independent work (usually the day after the start of reading period). Junior papers tend to be more substantial in both length and content than papers written during junior seminars.

Junior papers are evaluated primarily by the adviser; a second reader is not required for junior independent work. However, if the primary adviser is outside the mathematics department (which is allowed, supported, and indeed occurs frequently), a second reader should be arranged within the mathematics department. (This is to help ensure uniformity in the evaluation process.) Junior papers in mathematics are evaluated as per the guidelines on p. 7.
Senior Theses

Senior theses are expected and tend to be more in-depth, more original in exposition/content, and more substantial than junior papers. However, there are no formal length requirements. Some of the best senior theses in mathematics over the years have been under 10 pages, while others have been over 100 pages, although about 20-40 pages when single-spaced seems typical. Senior theses are evaluated on the same basic criteria as junior papers (see p. 7), but naturally more originality and imagination in the exposition and content are expected in senior theses.

The senior thesis is due by the university set deadline (usually, the start of the spring reading period). The thesis is evaluated by both the adviser and a second reader; thus, a second reader should be arranged at least a couple of weeks before the due date of the senior thesis (ideally much sooner). The thesis is "defended" to both the adviser and the second reader during a thesis defense that occurs a couple of days after Dean's date, during the final examination period. The defense consists of a short (about 25 minute) presentation by the student (see the exposition suggestions on pp. 9 and 10 below), followed by questions from the readers pertaining to the content of the thesis. Two grades are given: one for the quality of the presentation and defense (to emphasize the importance of mathematical communication), and one for the thesis itself.

The primary adviser for the senior thesis can be a university faculty member outside the mathematics department (which is allowed, supported, and indeed occurs frequently). In this case, the second reader should be arranged within the mathematics department. (This is to help ensure uniformity in the evaluation process.)

Senior thesis grades, together with grades in mathematics courses, are also taken into account when determining departmental honors.

A LaTeX template for senior theses is available at

http://cosmology.princeton.edu/cosmology/computing/PrincetonThesis.cls

Bound copies of all undergraduate mathematics theses are in the Lewis Science Library.
Finding an adviser

Some of the best advice for finding an adviser comes from those who have recently done so. The following advice is taken from the Princeton University Math Club’s *Guide for Math Students* and is drawn from the experience of recent math majors.

“Starting a thesis or JP requires two major steps: choosing an adviser and choosing a topic. In the overwhelming majority of cases, the former comes first. Choosing your adviser carefully is important. Your adviser’s style and the compatibility between the two of you will deeply influence the quality of your experience ... you should consult older students and the faculty academic advisers to figure out which professors might be a good fit for you in terms of research interests and advising style. As always, you will have to be proactive to ensure your experience is all that it can be; ask your peers many questions: about the frequency and content of meetings, the expectations for an undergraduate project (too low? too high?), the level of preparation expected, and so on.

Once you have an adviser, you will still need to find an effective way to work together. Sometimes, this will come naturally; that’s especially likely if your adviser often takes undergraduate students. Be that as it may, figure out how often meeting with your adviser is productive; once a week is standard, but some professors prefer biweekly meetings. Even if you have nothing to report, meeting with your adviser helps both of you stay in touch with the project and is an integral part of the research experience. You will also want to prepare for your meetings so as to get the most out of them. While meetings will be your primary interaction with your adviser, emails and other day-to-day interactions can be nearly as important. These generally take the form of questions and, here again, you will need to figure out how to make them work best for you. Experience is, for better or for worse, the only real way to learn how to do this. Finally, whoever your adviser is, you will benefit from making friends with their graduate students and postdocs (short for “postdoctoral fellows,” researchers who have recently obtained Ph.D.’s); they can serve as secondary advisers who can help you on a day-to-day basis—and share their experiences with early career research.”

This and other potentially useful advice from recent math majors can be found at [http://blogs.princeton.edu/mathclub/guide/research/](http://blogs.princeton.edu/mathclub/guide/research/)

The primary adviser for a junior paper or senior thesis can be a university faculty member outside the mathematics department (which is allowed, supported, and indeed occurs frequently). In this case, the second reader should be arranged within the mathematics department. (Again, this is simply to help ensure uniformity in the evaluation process.)
Timeline and Important Deadlines

Junior Seminars and JP’s

First week of semester: Attend organizational meeting for junior seminars

Second week: Confirm participation in a junior seminar with the undergraduate program office. If not participating in a junior seminar, notify both the undergraduate office and the faculty junior adviser that you are writing a junior paper; turn in completed form listing your JP topic and adviser (to be signed by your JP adviser).

University deadline for junior independent work: All JPs and seminar papers due. Please submit a .pdf of the paper to the undergraduate program office.

Senior Theses

Fourth week of fall semester: First thesis report due, including the name of thesis adviser and preliminary description of thesis topic. The undergraduate program office will email the report form.

Fourth week of spring semester: Thesis progress report due, including a request for the name of a second reader.

Tenth week of spring semester (two weeks before reading period): It is recommended that a draft of the thesis be given to the two readers.

First day of reading period: Thesis due. A bound copy of the thesis must be delivered to the undergraduate program office. A .pdf file must also be emailed to the undergraduate office. Final copies should also be given to your readers.

Reading period: Confirm scheduling of departmental exam and any special requests (media needs, etc.).

Senior departmental exams: usually first two days of exam period

Extensions: Extensions to the final deadlines are rare and granted only for exceptional circumstances. Requests for extensions must be discussed with the junior seminar leader, JP or thesis adviser, the faculty junior and senior advisers in the mathematics department, and the appropriate college dean(s).
Evaluation of Independent Work

The final grade for independent work is assigned by the seminar leader or JP adviser (with the second reader, if applicable), in the case of junior independent work, and jointly by the thesis adviser and second reader, in the case of the senior thesis. In the case of the senior thesis, two grades are assigned: one for the quality of the presentation and defense, and one for the thesis itself.

The following is a guide for the assignment of grades for independent work. These are only recommendations. The JP and theses readers have full discretion in the assignment of final grades. Questions about the quality of independent work or an adviser’s standard’s for grades should be asked well before the final due dates.

Suggested Standards for grading written independent work

Basic criteria for the evaluation written independent work are:

1. Imagination
2. Sophistication of material presented
3. Quality of exposition
4. Amount of learning and effort.

Original mathematical research, though hoped for, is not expected. Imagination might be indicated by the author’s point of view toward the subject or other contributions to the proofs of theorems presented. Clarity and mathematical precision are important factors in the quality of exposition.

A+ This grade is reserved for work that is exceptional in all four qualities: normally such would involve publishable original results or a significant redevelopment of known results. It should also be well written.

A The paper succeeds evidently and impressively with respect to the four criteria and has no serious flaws.

A- The paper succeeds competently with respect to the four criteria and has no serious flaws.

B+ The paper fails to meet all of these criteria, but still meets a number of them significantly.

B The paper fails to meet all of these criteria, but still meets at least one of them significantly and is a competent effort on all fronts.
**B-** The paper does not meet all the standard criteria to the expected extent and does not meet any of them to an impressive extent, but is a competent effort.

**C+** A paper that has no distinction other than a clear exposition of moderately sophisticated material, but does exhibit a reasonable amount of learning and effort.

**C, C-** A paper that is passable but distinctly inferior, i.e., little effort or major flaws.

**D** For senior theses, D implies lack of endorsement for graduation but does not prevent it.

**F** The paper fails on all points and indicates little or no effort. An F on a senior thesis prevents the student from graduating.
Some suggestions for writing a good mathematics paper

Make sure to motivate the subject matter of your paper, by briefly describing the history of the topic and some specific reasons why the topic is important and interesting. The introduction should be a teaser that attracts the reader into your world and this paper.

State definitions and theorems clearly. Good grammar and clean suggestive notation and terminology are essential. Do your best to define the least amount of notation and terminology necessary to state your main theorem(s). Make sure to give examples throughout to help comprehension. For more detailed, technical, or lengthy arguments, try to give the global overall picture first so that the reader is not as afraid to delve into the details. Mathematicians frequently judge arguments by their "elegance", so do your best to be elegant! Complete arguments are also appreciated!

Some suggestions for giving a good mathematics lecture

1. Make eye contact with members of the audience regularly.

   Too often the speaker is talking only to the blackboard, or only to one person, and this should be avoided! Everyone should feel part of the experience.

2. Project your voice and enunciate.

   Speaking in your normal voice will sound like mumbling when you lecture, so make sure to project your voice so even those sitting in the back can hear you clearly.

3. Write legibly and large enough.

   Use blackboards wisely and in an organized manner, typically from top to bottom and then left to right. Draw lines on the blackboard to separate writing from different trains of thought.

   Extremely important!: state definitions and theorems in full. Saying something orally doesn't mean that you don't need to write it down too! A good rule of thumb on what to write on the board: if audience members were to take notes by simply copying down what you wrote on the blackboard (without listening to a word you said), they should still be able to understand and appreciate the lecture if they look at their notes years from now.
4. (A more advanced skill) Don't be shy to repeat and re-repeat statements to drive home important points.

   Feel free to underline or put boxes around important formulas or concepts! Colored chalk can also help in creating appropriate emphasis. Feel free to be creative.

   Learn to be able to continue to talk while writing, erasing, and moving blackboards, so that this can be done when needed.

   However, on more difficult points, or every once in a while, be sure to take an appropriate pause, so that audience members can have time to digest or ask questions.

5. A good lecture should typically contain at least one theorem, one example, and one joke!
Resources for independent work

1. Your adviser.

The best resource for learning about doing mathematical research is someone who does it as a profession. Do not be afraid to ask lots of questions!

2. The library.

Lewis library contains an exceptional collection of mathematical books and journals (many of the latter are also online). Browsing often leads to unexpected connections.

The library also contains bound copies of all senior theses. These provide examples of what to do (and not do!) when writing mathematics.

3. The internet.

There are many helpful online resources: MathSciNet (which contains reviews of most published papers in mathematics), electronic versions of journals and books, preprint servers, webpages of mathematicians. If you need help with any of these, the Lewis library staff can help.

4. Other students.

Talking to other students about their experience doing mathematical research can be both helpful and stimulating.

There is a reason mathematicians spend so much time at conferences: explaining one's work and hearing others explain theirs is a big part of doing mathematics. It can help clarify ideas and provide new insight.

The Princeton Undergraduate Mathematics Club has compiled advice from the experiences of many other students:

http://blogs.princeton.edu/mathclub/guide/research/
5. The junior and senior departmental advisers.

   Talk to them about any problems or difficulties, no matter how small, before they become big.

6. The undergraduate program office.

   The source for all questions about procedures and deadlines.

7. The Writing Center.

   The Writing Center, located in Whitman College, offers an array of options for help with writing junior paper and senior theses:

   http://www.princeton.edu/writing/center/