

**Analysis and Applications:
A Conference in Honor of Elias M. Stein
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Speaker: Peter Jones (Yale University)

Date/Time: Wednesday, May 18, 2011 / 10:15-11:15 am

Talk Title: Product Formulas for Measures and Applications to
Analysis and Geometry

Abstract:

We will discuss geometry of Lebesgue measurable sets and differentiability of Lipschitz functions. The starting point is elementary product formalisms for positive measures, due to R. Fefferman, C. Kenig, and J. Pipher. We will give some background where there have been previous applications to analysis and geometry. Most of the talk will be devoted to joint work with Marianna Csörnyei. The new result concerns Lebesgue measurable sets E of small Lebesgue measure (in any dimension). The set E can be decomposed into a bounded number of sets with the property that each (sub)set has a nice "tangent cone". Roughly speaking each subset has very small intersection with any Lipschitz curve whose tangent vector (to that curve) always lies inside a fixed cone. This had been proven in dimension two by Alberti, Csörnyei, and Preiss by using special, two dimensional combinatorial arguments. The main technical result needed in our work is a d dimensional, measure theoretic version of (a geometric form of) the Erdős-Szekeres theorem. (The discrete form of E-S is known only in $d = 2$.) In what is perhaps a small surprise, certain ideas from random measures can be used effectively in the deterministic setting. Our result yields strong results on Lipschitz functions: For any Lebesgue null set E in d dimensions, there is a Lipschitz mapping of Euclidean d space to itself, that is nowhere differentiable on E . (The Rademacher theorem is sharp.) This result is best possible.