Optimal Transportation and Ricci Curvature for Metric Measure Spaces

We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M,d,m), based on convexity properties of the relative entropy $Ent(.|m)$. For Riemannian manifolds, $Curv(M,d,m) \geq K$ if and only if $Ric_M \geq K$ on M. For the Wiener space, $Curv(M,d,m) = 1$.
One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence.

Moreover, we introduce a curvature-dimension condition $\mathrm{CD}(K,N)$ being more restrictive than the curvature bound $Curv(M,d,m) \geq K$. For Riemannian manifolds, $\mathrm{CD}(K,N)$ is equivalent to $\mathrm{Ric}_M(\xi,\xi) \geq K \cdot |\xi|^2$ and $\dim(M) \leq N$. Condition $\mathrm{CD}(K,N)$ implies sharp version of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct canonical Dirichlet forms with Gaussian upper and lower bounds for the corresponding heat kernels.