Speaker: Chris Skinner, Princeton Univeristy and Univ. of Michigan
Title: L-values and Congruences

Abstract:

Consider the series $\Delta(q)$ defined by

$$\Delta(q) = q \prod_{n=1}^{\infty} (1 - q^n)^2 4 = \sum_{n=1}^{\infty} \tau(n) q^n.$$

The famous Ramanujan congruence

$$\tau(p) \equiv 1 + p^1 1 \pmod{691}, \tag{*}$$

for every prime prime p, is a consequence of the fact that the prime 691 appears in the denominator of $\zeta(-11)$, where $\zeta(s)$ is the Riemann zeta function. The congruence (*) in turn implies the non-triviality of a piece of the class group of Q(ζ_{691}), the cyclotomic field obtained by adjoining a 691st root of unity to Q.

I will begin by explaining the circle of ideas - due to Ribet, Mazur, and Wiles - that explains (*), its consequences, and their generalizations to other values of $\zeta(s)$ and of Dirichlet L-series. Then I will discuss how these methods have been extended, connecting values of L-functions of elliptic curves and modular forms to generalizations of (*) and even to orders of various Galois cohomology groups.